Designation: E111 – 17

Standard Test Method for Young's Modulus, Tangent Modulus, and Chord Modulus[1]

This standard is issued under the fixed designation E111; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

This standard has been approved for use by agencies of the U.S. Department of Defense.

1. Scope*

1.1 This test method covers the determination of Young's modulus, tangent modulus, and chord modulus of structural materials, see Fig. 1. This test method is limited to materials in which and to temperatures and stresses at which creep is negligible compared to the strain produced immediately upon loading and to elastic behavior.

1.2 Because of experimental problems associated with the establishment of the origin of the stress-strain curve described in 8.1, the determination of the initial tangent modulus (that is, the slope of the stress-strain curve at the origin) and the secant modulus are outside the scope of this test method.

1.3 The values stated in SI units are to be regarded as standard. No other units of measurement are included in this standard.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety, health, and environmental practices and determine the applicability of regulatory limitations prior to use.*

1.5 *This international standard was developed in accordance with internationally recognized principles on standardization established in the Decision on Principles for the Development of International Standards, Guides and Recommendations issued by the World Trade Organization Technical Barriers to Trade (TBT) Committee.*

2. Referenced Documents

2.1 *ASTM Standards:*[2]

E6 Terminology Relating to Methods of Mechanical Testing
E8/E8M Test Methods for Tension Testing of Metallic Materials
E9 Test Methods of Compression Testing of Metallic Materials at Room Temperature
E21 Test Methods for Elevated Temperature Tension Tests of Metallic Materials
E83 Practice for Verification and Classification of Extensometer Systems
E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods
E1012 Practice for Verification of Testing Frame and Specimen Alignment Under Tensile and Compressive Axial Force Application

2.2 *General Considerations*—While certain portions of the standards and practices listed are applicable and should be referred to, the precision required in this test method is higher than that required in general testing.

3. Terminology

3.1 *Definitions:* Terms common to mechanical testing.

3.1.1 The definitions of mechanical testing terms that appear in Terminology E6 apply to this test method. These terms include initial tangent modulus, secant modulus, gauge length, yield strength, tensile strength, stress-strain diagram, and extensometer.

3.1.2 The terms accuracy, precision, and bias are used as defined in Practice E177.

3.1.3 In addition, the following common terms that appear in the Terminology E6 apply to this test method.

3.1.4 *chord modulus*—the slope of the chord drawn between any two specified points on the stress-strain curve below the elastic limit of the material.

3.1.5 *elastic limit [FL^2], n*—the greatest stress that a material is capable of sustaining without any permanent strain remaining upon complete release of the stress.

3.1.5.1 *Discussion*—Due to practical considerations in determining the elastic limit, measurements of strain using a small force, rather than zero force, are usually taken as the initial and final reference.

3.1.6 *indicated temperature, n*—the temperature indicated by a temperature measuring device using good pyrometric practice.

[1] This test method is under the jurisdiction of ASTM Committee E28 on Mechanical Testing and is the direct responsibility of Subcommittee E28.04 on Uniaxial Testing.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

***A Summary of Changes section appears at the end of this standard**

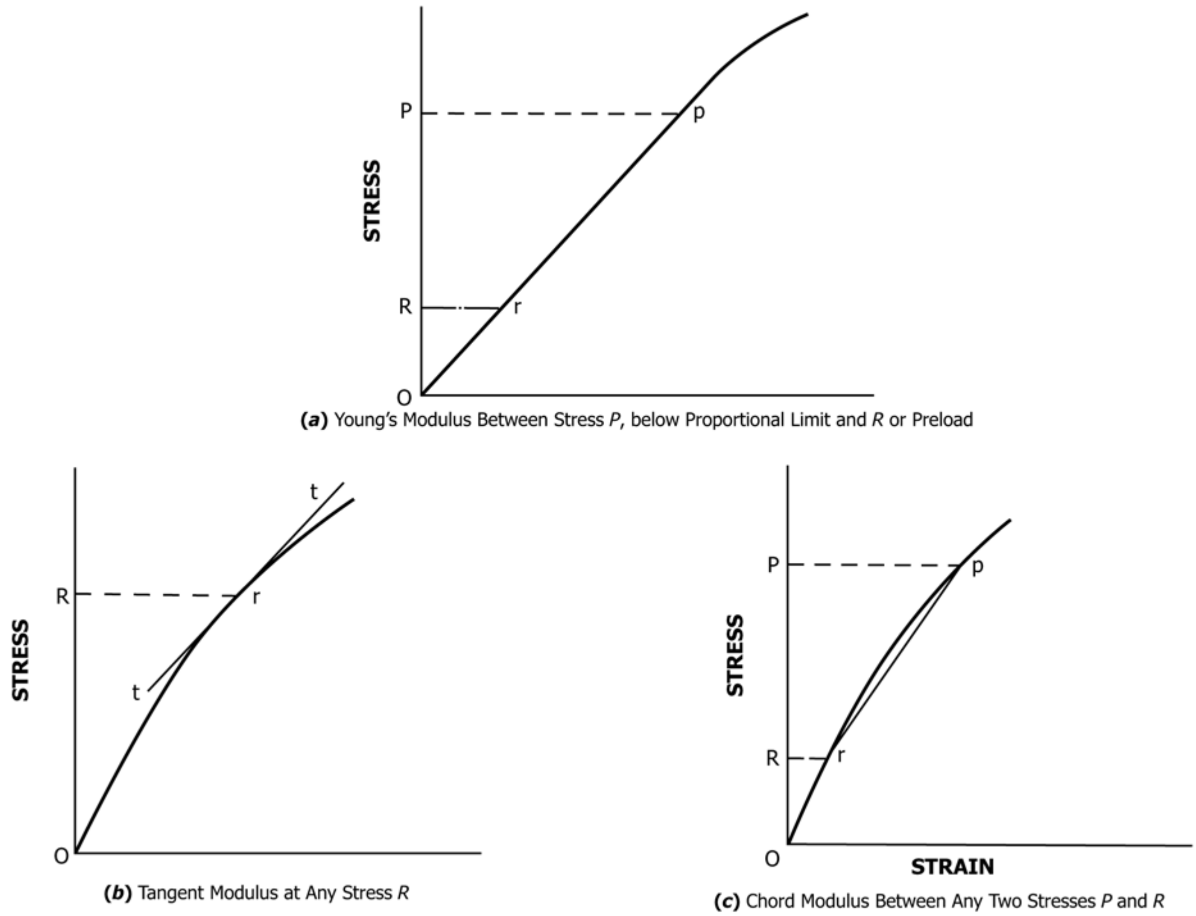

FIG. 1 Stress-Strain Diagrams Showing Straight Lines Corresponding to (*a*) Young's Modulus, (*b*) Tangent Modulus, and (*c*) Chord Modulus

3.1.7 *nominal temperature, n*—the intended test temperature.

3.1.8 *proportional limit [FL^2], n*—the greatest stress that a material is capable of sustaining without deviation from proportionality of stress to strain (Hooke's law).

3.1.9 *tangent modulus*—the slope of the stress-strain curve at any specified stress or strain.

3.1.10 *Young's modulus*—the ratio of tensile or compressive stress to corresponding strain below the proportional limit.

4. Summary of Test Method

4.1 A uniaxial force is applied to the test specimen and the force and strain are measured, either incrementally or continuously. The axial stress is determined by dividing the indicated force by the specimen's original cross-sectional area. The appropriate slope is then calculated from the stress-strain curve, which may be derived under conditions of either increasing or decreasing forces (increasing from preload to maximum applied force or decreasing from maximum applied force to preload).

5. Significance and Use

5.1 The value of Young's modulus is a material property useful in design for calculating compliance of structural materials that follow Hooke's law when subjected to uniaxial loading (that is, the strain is proportional to the applied force).

5.2 For materials that follow nonlinear elastic stress-strain behavior, the value of tangent or chord modulus is useful in estimating the change in strain for a specified range in stress.

5.3 Since for many materials, Young's modulus in tension is different from Young's modulus in compression, it shall be derived from test data obtained in the stress mode of interest.

5.4 The accuracy and precision of apparatus, test specimens, and procedural steps should be such as to conform to the material being tested and to a reference standard, if available.

5.5 Precise determination of Young's modulus requires due regard for the numerous variables that may affect such determinations. These include (*1*) characteristics of the specimen such as orientation of grains relative to the direction of the stress, grain size, residual stress, previous strain history, dimensions, and eccentricity; (*2*) testing conditions, such as alignment of the specimen, speed of testing, temperature, temperature variations, condition of test equipment, ratio of error in applied force to the range in force values, and ratio of error in extension measurement to the range in extension values used in the determination; and (*3*) interpretation of data (see Section 9).

5.6 When the modulus determination is made at strains in excess of 0.25 %, correction shall be made for changes in cross-sectional area and gauge length, by substituting the instantaneous cross section and instantaneous gauge length for the original values.

5.7 Compression results may be affected by barreling (see Test Methods E9). Strain measurements should therefore be made in the specimen region where such effects are minimal.

6. Apparatus

6.1 *Dead Weights*—Calibrated dead weights may be used. Any cumulative errors in the dead weights or the dead weight loading system shall not exceed 0.1 %.

6.2 *Testing Machines*—In determining the suitability of a testing machine, the machine shall be calibrated under conditions approximating those under which the determination is made. Corrections may be applied to correct for proven systematic errors.

6.3 *Loading Fixtures*—Loading fixtures shall be properly designed and maintained. The allowable bending as defined in Practice E1012 shall not exceed 5 %.

NOTE 1— Grips and other devices for obtaining and maintaining axial alignment are shown in Test Methods E8/E8M and E9. Procedures for verifying the alignment are described in detail in Practice E1012.

6.4 *Extensometers*—Class B-1 or better extensometers as described in Practice E83 shall be used. Corrections may be applied for proven systematic errors in strain and are not considered as a change in class of the extensometer. Either an averaging extensometer or the average of the strain measured by at least two extensometers arranged at equal intervals around the cross section shall be used. If two extensometers are used on other than round sections, they shall be mounted at ends of an axis of symmetry of the section. If a force-strain recorder, strain-transfer device, or strain follower is used with the extensometer, they shall be calibrated as a unit in the same manner in which they are used for determination of Young's modulus. The gauge length shall be determined with an accuracy consistent with the precision expected from the modulus determination and from the extensometer.

NOTE 2—The accuracy of the modulus determination depends on the precision of the strain measurement. The latter can be improved by increasing the gauge length. This may, however, present problems in maintaining specimen tolerances and temperature uniformity.

6.5 *Furnaces or Heating Devices*—When determining Young's modulus at elevated temperature, the furnace or heating device used shall be capable of maintaining a uniform indicated temperature in the reduced section of the test specimen so that a variation of not more than ±1.5°C for nominal temperatures up to and including 900°C, and not more than ±3.0°C for temperatures above 900°C, occurs. (Heating by self-resistance shall not be used.) Minimize indicated temperature variations and control changes within the allowable limits. An instrumented sample representative of the real test may be used demonstrate that the setup meets the above capabilities.

NOTE 3—Differences in thermal expansion between specimen and extensometer parts can cause significant errors in apparent strain.

6.6 *Low-Temperature Baths and Refrigeration Equipment*—When determining Young's modulus at temperatures below room temperature, an appropriate low-temperature bath or refrigeration system shall be used to maintain the specimen at the nominal temperature during testing. For a low-temperature bath, the lower tension rod or adapter may pass through the bottom of an insulated container and be welded or fastened to it to prevent leakage.

NOTE 4—For nominal temperatures to about – 80°C, chipped dry ice that cools an organic solvent such as ethyl alcohol in the low-temperature bath is suitable. Other organic solvents having lower solidification temperatures, such as methylcyclohexane or isopentane, cooled with liquid nitrogen are useful at temperatures lower than – 80°C. Liquid nitrogen can be used to achieve a nominal temperature of – 196°C. Lower nominal temperatures are possible with liquid hydrogen and liquid helium, with special containers or cryostats to minimize heat leakage and to permit efficient use of these coolants. Liquid hydrogen can produce explosive mixtures of hydrogen gas and air. If refrigeration equipment is used to cool the specimens with air as the cooling medium, it is desirable to have forced air circulation to provide uniform cooling.

6.6.1 At low temperatures, when using a coolant bath, immersion-type extensometers should be used.

6.7 Temperature measuring, controlling, and recording instruments shall be calibrated periodically against a secondary standard, such as a precision potentiometer. Lead-wire error should be checked with the lead wires in place as they normally are used.

7. Test Specimens

7.1 *Selection and Preparation of Specimens*—Special care shall be taken to obtain representative specimens that are straight and uniform in cross section. If straightening of the material for the specimen is required, then resultant residual stresses shall be removed by a subsequent stress relief annealing procedure that shall be reported with the test results.

7.2 *Dimensions*—The specimen length (and fillet radius in the case of tension specimens) should be greater than the minimum requirements for general-purpose specimens. In addition, the ratio of length to cross section of compression specimens should be such as to avoid buckling (see Test Methods E9).

NOTE 5—For examples of tension and compression specimens, see Test Methods E8/E8M and E9.

7.3 For tension specimens, the center lines of the grip sections and of the threads of threaded-end specimens shall be concentric with the center line of the gauge section within close tolerances in order to obtain the degree of alignment required. If pin-loaded sheet-type specimens are used, the centers of the gripping holes shall be not more than 0.005 times the width of the gauge section from its center line. For sheet-type specimens, small tabs or notches for attaching the extensometer may be used.

NOTE 6—The effect of eccentric loading can be illustrated by calculating the bending moment and stress thus added. For a standard 12.5-mm diameter specimen, the stress increase is 1.5 % for each 0.025 mm of eccentricity. This error increases to about 2.5 % per 0.025 mm for a 9-mm diameter specimen and to about 3.2 % per 0.025 mm for a 6-mm diameter specimen.

7.4 The length of the reduced section of tension specimens shall exceed the gauge length by at least twice the diameter or twice the width. The length of compression specimens shall be in accordance with Test Methods E9, and all specimens shall have a uniform cross-sectional area throughout the gauge length.

7.4.1 If a general-purpose tension specimen such as those shown in Test Methods E8/E8M, having a small amount of taper in the reduced section is used, the average cross-sectional area for the gauge length should be used in computing stress.

7.5 For compression specimens, the ends shall be flat, parallel and perpendicular to the lateral surfaces as specified in Test Methods E9.

7.6 The specimen shall be free of residual stresses. The specimen may be subjected to an annealing procedure to relieve the residual stresses. If the intent of the test is to verify the performance of a product, the annealing procedure may be omitted. Report the condition of the material tested, including any annealing procedure.

NOTE 7—An annealing procedure at $T_m/3$ for 30 min to relieve the stresses in the material (where T_m is the melting point of the material in K) has been used successfully.

8. Procedure

8.1 Measurements shall be made from a small force or preload, known to be high enough to minimize extensomer output errors, to some higher applied force, still within either the proportional limit or elastic limit of the material. For linearly elastic materials, the slope of the straight-line portion of the stress-strain curve shall be established between the preload and the proportional limit to define Young's modulus. If the actual stress-strain curve is desired, this line may appropriately be shifted along the strain axis to coincide with the origin. For nonlinearly elastic materials the tangent or chord modulus may be established between the appropriate stress values on the stress strain curve.

NOTE 8—For most loading systems and test specimens, effects of backlash, specimen curvature, initial grip alignment, etc., introduce significant errors in the extensometer output when applying a small force to the test specimen.

8.2 *Measurement of Specimens*—Measure specimen dimensions at the ends of the gauge length and at least at one intermediate location to within 1 % accuracy.

8.3 *Alignment*—Ensure as nearly axial loading as possible. The strain increments between the initial-load and the final-load measurement on opposite sides of the specimens should not differ from the average by more than 3 %.

8.4 *Soaking Time of Specimens at Testing Temperature*—After the specimen to be tested has reached the nominal temperature, maintain the specimen at the nominal temperature for a sufficient length of time to attain equilibrium conditions of the specimen and extensometer before applying force. Report the time to attain the nominal temperature and the time at the nominal temperature before applying force.

NOTE 9—The recommended soak time at the nominal temperature is 1 hour per 25 mm (1 hour/inch) of specimen thickness or diameter. If the temperature of the system is not uniform by the time loading of the specimen is started, variations in thermal expansion will be reflected in the modulus line. Furthermore, fluctuations in temperature of the extensometer extensions during testing which result from cycling of the furnace temperature or changes in the level of the cooling bath can also affect the slope of the modulus line.

8.5 *Speed of Testing*—The speed of testing shall be low enough that thermal effects of adiabatic expansion or contraction are negligible and that accurate determination of force and extension is possible, yet the speed shall be high enough that creep will be negligible. In loading with dead weights, avoid temporary overloading due to inertia of the weights. The strain rate should be reported.

8.6 *Number of runs*—A minimum of three runs should be made for each specimen. Exercise care to not exceed the proportional limit in the case of Young's modulus, and the elastic limit in the case of the tangent or chord modulus. Report each of the three values or the average along with the method for determining them.

8.6.1 Young's modulus, tangent modulus, or chord modulus for a given specimen may be determined along with yield strength and tensile strength using a single loading cycle. If modulus values are determined this way, report that only one loading cycle was used. Three cycles within the elastic region as recommended in 8.6, may be used to determine the modulus, before straining the specimen into the plastic range to determine yield and tensile strengths.

8.7 *Temperature Control*—Keep the variation of the indicated temperature from the actual tempertaure as small as is practical through good practice and precise control. The average indicated temperature over the specimen gauge length shall not deviate from the nominal temperature by more than ±2°C. In elevated-temperature tests, indicated temperature variations along the gauge length of the specimen shall not exceed the following limits: up to and including 900 ± 1.5°C, above 900 ± 3.0°C. (See 6.5.) The test must be performed with the same setup and under similar conditions as those of the instrumented test described in 6.5. Temperature changes should be minimized while making strain measurements.

NOTE 10—The actual temperatures can vary more than the indicated temperatures. Temperature changes during the test, within the allowable limits, can cause significant strain errors due to differences in thermal expansion of the test specimen and extensometer parts.

8.8 In low-temperature testing in which the bath is cooled with dry ice or in which a refrigeration system is used, the indicated temperature of the medium around the specimen shall be maintained at temperatures within 1.5°C of the specified temperature. Bath indicated temperatures or the temperature of circulating air from a refrigeration system may be measured with a thermocouple or a suitable thermometer. If the specimen is submerged in a bath at the boiling point of the bath, allow sufficient soaking time (see Note 9) to provide equilibrium conditions. Specimens tested in boiling liquids shall meet the temperature-control requirements specified in 8.7.

NOTE 11—The boiling point of a commercial liquid gas may not be the same as the published temperature for the pure liquid gas.

8.9 *Temperature Measurement*—The method of temperature measurement shall be sufficiently sensitive and reliable to ensure that the temperature of the specimen is within the limits specified in 8.7 and 8.8.

NOTE 12—Thermocouples in conjunction with potentiometers or millivolt meters are generally used to measure temperatures. A discussion of temperature measurement and the use of thermocouples is given in Test Methods E21.

9. Interpretation of Data

9.1 When the modulus determination is made at strains in excess of 0.25 %, correct for changes in cross-sectional area and gauge length by substituting the instantaneous cross section and instantaneous gauge length for the original values.

9.2 *Graphical Data Method:*

9.2.1 If a plot of force-versus-extension (force versus elongation) is obtained by graphically, compute the value for Young's modulus is obtained by determining the slope of the line for forces less than the force corresponding to the proportional limit. Choose the lower force point consistent with the limitations set forth in 8.1. Compute Young's modulus from the force increment and corresponding extension increment, between two points on the line as far apart as possible, by using Eq 1:

$$E = \left(\frac{\Delta_p}{A_o}\right) / \left(\frac{\Delta_c}{L_o}\right) \quad (1)$$

where:

Δ_p = force increment,
A_o = original cross-sectional area,
Δ_c = extension increment, and
L_o = original gauge length.

9.2.2 The report should include an estimate of the precision of the reported value of Young's modulus based on the summation of the precisions of the respective values.

NOTE 13—The precision of the value obtained for Young's modulus will depend upon the precision of each of the values used in the calculation

9.3 *Numerical Data Method:*

9.3.1 If the force-versus-extension data are obtained in numerical form, compute the Young's modulus by the method of least squares.

NOTE 14—The errors introduced by plotting the data and fitting graphically a straight line to the experimental points are reduced by determining Young's modulus as the slope of the straight line fitted to the appropriate data. This method also permits statistical study of the data and therefore an evaluation of the variability of the modulus within the stress range employed.

9.3.2 Calculate the Young's modulus using Eq 2:

$$\text{Young's modulus, } E = \frac{\sum (XY) - K\bar{X}\bar{Y}}{\sum X^2 - K\bar{X}^2} \quad (2)$$

where:

Y = applied axial stress, and
X = corresponding strain. = .
$\bar{Y}$ = $\sum Y/K$= average of Y values
$\bar{X}$ = $\sum X/K$= average of X value
K = number of X,Y data pairs and $\sum$ = sum from 1 to K.

In terms of the measured force P_i and measured original cross-sectional area A_o and gauge length L_o,

$$X = \frac{\Delta_c}{L_o}$$

$$Y = \frac{\Delta_p}{A_o}$$

Calculate the coefficient of determination, r^2, which indicates the goodness of fit achieved in a single test using Eq 3:

$$r^2 = \frac{\left[\sum XY - \frac{\sum X \sum Y}{K}\right]^2}{\left[\sum X^2 - \frac{(\sum X)^2}{K}\right]\left[\sum Y^2 - \frac{(\sum Y)^2}{K}\right]} \quad (3)$$

The values of r^2 should be close to 1.00 (see Table 1).

TABLE 1 Fitting of Straight Lines Coefficient of Variation of Slope (Percent) (V_1)

Data Pairs (*K*)	Sample Correlation Coefficients (*r*)				
	0.90000	0.99000	0.99900	0.99990	0.99999
3	±48.4	±14.2	±4.47	±1.41	±0.447
5	27.9	8.22	2.58	0.816	0.258
10	17.1	5.03	1.58	0.500	0.158
20	11.4	3.35	1.05	0.333	0.105
30	9.1	2.69	0.84	0.267	0.084
50	6.9	2.05	0.64	0.204	0.064
100	4.8	1.44	0.45	0.142	0.045

Calculate the coefficient of variation of the slope using Eq 4 (see Table 1 for representative values):

$$V_1 = 100\sqrt{\frac{\frac{1}{r^2} - 1}{K - 2}} \quad (4)$$

where:

V_1 = coefficient of variation, %

NOTE 15—Under normal circumstances the coefficient of variation will not be larger than 2 %; however with care, values less than 0.5 % have been achieved in aluminum alloys.

9.3.3 To establish confidence intervals for the regression line for Young's modulus the Eq 5 may be used:

$$\pm I = tV_1 \tag{5}$$

where:

I = percent of slope confidence interval,
V_1 = coefficient of variation, expressed in percent (see 9.3), and
t = $t-$statistic from standard tables at $K-2$ degrees of freedom and confidence level selected.

Table 2 gives an example of representative values calculated using a 95 % confidence interval.

9.3.4 In the case of nonlinear elastic materials, the stress-strain curve may be obtained by fitting a polynomial approximation to the force-versus-extension or force-versus-strain data pairs. Compute the chord modulus between two specified sets of data pairs below the elastic limit on the fitted polynomial curve. Choose the lower of the two sets of data pairs consistent with the limitations in 8.1.

9.3.5 The tangent modulus may be determined by any method the user deems reasonable, but the method shall be reported.

NOTE 16—One method for computing the tangent modulus is to evaluate the value of the first derivative of the polynomial fit at the strain of interest.

10. Report

10.1 Report the following information:

10.1.1 *Specimen Material*—Specimen material, alloy, heat treatment, mill batch number, grain direction, and other relevant material information.

10.1.2 *Specimen Configuration*—Sketch of the specimen configuration or reference to the specimen drawing.

10.1.3 *Specimen Dimensions*—Actual measured dimensions for the specimen.

TABLE 2 Fitting of Straight Lines for 95 % Confidence Interval Percentage Values of Slope Confidence Interval (*I*)

Data Pairs (*K*)	*t*-Statistic	Sample Correlation Coefficients (*r*)			
		0.99000	0.99900	0.99990	0.99999
3	12.71	±180	±56.8	±17.9	±5.7
5	3.182	26.2	8.2	2.6	0.8
10	2.306	11.6	3.6	1.2	0.4
20	2.101	7.0	2.2	0.7	0.2
30	2.048	5.5	1.7	0.5	0.17
50	2.011	4.1	1.29	0.41	0.129
100	1.984	2.8	0.89	0.28	0.089

10.1.4 *Test Fixture*—Description of the test fixture or reference to fixture drawings.

10.1.5 *Testing Machine and Extensometers*—Manufacturer, model, serial number, and force range of the testing machine and the extensometers.

10.1.6 *Speed of Testing*—Test rate and mode of control.

10.1.7 *Temperature*—Nominal temperature, time to attain nominal temperature and time at nominal temperature before applying force.

10.1.8 *Stress-Strain Diagram*—Stress-strain diagram with scales, specimen number, test data, rate, and other pertinent information.

10.1.9 *Young's Modulus, Tangent Modulus, Chord Modulus*—Modulus value and the method used to determine the value in accordance with Section 9.

11. Precision and Bias

11.1 *Precision*—The following parameters are reported to impact upon the precision of this test method:

11.1.1 Characteristics of the specimen such as orientation of grains relative to the axial stress, grain size, residual stress, previous strain history, dimensions, and eccentricity.

11.1.2 Testing conditions such as alignment of the specimen, speed of testing, temperature, temperature variations, conditions of test equipment, ratio of error in force to the range in force values, and ratio of error in extension measurement to the range in extension values.

11.1.3 Interpretation of data such as whether graphical or digital data were taken, calibration of recording or data-logging device, number of data pairs used to obtain slope of stress-strain curve (see Table 1). One measure of the precision of Young's modulus is the confidence interval for the computed regression line as shown in 9.3.3.

11.2 *Bias*—A statement of bias of this test method requires reference standard values for one or more materials based on many measurements. Such standard reference values are presently not available.

NOTE 17—While a large amount of published data on Young's modulus of various materials are available in the open literature, it is unlikely that these data had been determined by using the exact procedure described in this test method. This will require interlaboratory test programs utilizing the procedures of this test method on various materials. Therefore, at the present time, the bias of the test method is unknown. However, calibration standards are available for testing machines and measuring devices.

12. Keywords

12.1 chord modulus; stress-strain diagram; tangent modulus; Young's modulus

SUMMARY OF CHANGES

Committee E28 has identified the location of selected changes to this standard since the last issue (E111 – 04(2010)) that may impact the use of this standard.

(1) The strain deviation method was removed.
(2) Some requirements were clarified and requirements were removed from notes.
(3) Some terms from Terminology E6 were added to the terminology section.